

# **Statics and stability for thin-walled shear beam with cross-section distortion**

**Robert Cybulski (s080836)**

.....

**Master's thesis**

Technical University of Denmark- DTU Byg

July 12, 2010

Project Advisors:

- Associate Prof. Leif Otto Nielsen, Department of Civil Engineering
- Prof. Jeppe **Jönsson**, Department of Civil Engineering

## Table of Contents

Abstract.....	4
Introduction.....	5
1. Notations and keywords used for the theory.....	5
2. The linear statics of TWB.....	6
a) Bending in the cross-section.....	6
<i>Example 1</i> .....	8
b) Timoshenko beam behavior.....	15
<i>Example 2</i> .....	16
c) St. Venant torsion of the walls .....	17
<i>Example 3</i> .....	17
d) TWB homogeneous differential equations.....	18
e) Polynomial solution and solution consisting of exponential modes.....	19
<i>Example 4</i> .....	20
Chapter 1- Interpolation matrix for TWB element.....	24
1. Consideration of TWB element .....	24
Chapter 2- FEM formulation of TWB element and structure for stability problems.....	26
1. The TWB element stiffness matrix .....	26
2. The TWB element stress-stabilization matrix under constant compression load. ....	27
3. The TWB element stress-stabilization matrix under in-plane bending.....	30
4. The stiffness matrix and the stress-stabilization matrix of the system.....	31
5. The eigenvalue solution .....	32
Chapter 3- FEM program for TWB structure with tests.....	33
1. Description of the “TWBSaS” .....	33
2. Results testing .....	33
a) Warping mode.....	33

b) Column buckling mode without shear consideration .....	35
c) Column buckling mode with shear consideration.....	37
d) Torsion buckling.....	40
e) In-plane bending .....	43
Chapter 4- Application of TWBSaS .....	46
1. Global column buckling and local plate buckling investigation.....	46
Conclusions .....	57
Bibliography.....	58
Software.....	58
Appendix 1.....	59
Appendix 2.....	64
Appendix 3.....	65
Appendix 4.....	66

**Abstract**

This Master's thesis deals with the thin-walled shear beam statics and stability with cross-section distortion. The linear statics based on reference [6] includes the bending in the cross-section of each wall as a Bernoulli beam, Timoshenko beam behavior (in-plane stretch, bending and shear of walls) with free torsion as a Saint-Venant torsion of the walls in the thin-walled beam's element direction. The material is elastic. The stability phenomenon includes formulation of the element stiffness matrix and the stress-stabilization matrix under constant compression load and under in-plane banding. Then the stiffness matrix and the stress-stabilization matrix on the system level with eigenvalue problem formulation are presented.

The purposes of this work are the following:

- to implement an example for theory presented in [6] due to better understanding of the linear statics formulation of the thin-walled beam,
- to present the solution of homogeneous differential equations in terms of polynomial solution and solution consisting of the exponential modes on arbitrary chosen thin-walled beam cross-sections,
- to extend theory in reference [6] by the buckling theory,
- to implement above theories into MatLab and create the program which can handle different cases of the thin-walled beam's buckling modes,
- to compare results given in the program with the existing theories.

The created program called "Thin-walled beam statics and stability" (TWBSaS) has been checked for the following cases of the buckling modes:

- the warping mode with I-profile cross-section,
- the column buckling mode with and without shear consideration with a single wall and I-profile cross-section under constant compression load,
- the torsion buckling mode with a cruciform cross-section under constant compression load,
- the mode caused by in-plane bending with a slender I-profile cross-section,
- the local buckling mode and distortional mode with hollow-core rectangular cross-section under constant compression load.

The software used for the present work is MatLab (numerical calculus) and Maxima (analytical calculus).