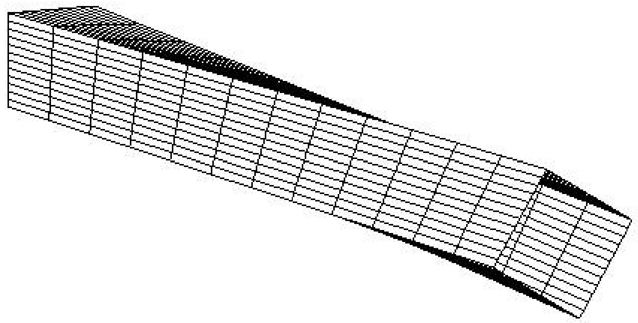
Beam Theory and Modelling of Distortion

Main report



Lotte Braad Sander, s071972

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DTU Civil Engineering
Department of Civil Engineering

Preface

This project is a master's thesis made at Technical University of Denmark in the fall semester 2012. The project corresponds to 32.5 ECTS points and is made at Department of Civil Engineering.

The project consists of this main report and a small Matlab program. The program is based on advanced thin walled beam elements which includes distortion and is tried developed in such a manner that it can be used by others. Some knowledge about the subject is required in order to be able to understand and modify the program. The program consists of multiple files and functions and a manual is therefore made. This manual is also a part of the project and describes the program in more details. The program can be found on the enclosed CD-ROM.

This master's thesis is made in close cooperation with Martin Mygind. The reports are made individually but the program and the Matlab manual are made as a common project. A big thank-you should be given to Martin Mygind for the cooperation during this project.

A thank to my supervisor Adjunct Michael Joachim Andreassen should also be given for help with Matlab programming and general advises through this project.

Finally I would like to thank my main supervisor Professor Jeppe Jönsson for the guidance, encouragement and advice he has provided me through this project. It has been enjoyable to have a supervisor that really was interested in the project and was ready to help us when needed.

Lotte Braad Sander, s071972

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Abstract

In this master's thesis beam elements made by thin-walled cross section have been analyzed. Since the use of thin walled structural elements have been increased during the last years because of the high strength and the effective use of material, more detailed calculations are needed. This project is based on the article "Distortional eigenmodes and homogeneous solutions for semi-discretized thin-walled beams" made by J. Jönsson and M.J. Andreassen and a further development is made such that the shear deformations also are included and no constraints due to shear are made. This is done in order to be able to include the Poisson's ratio in the calculations.

The formulation used is based on the generalized Euler-Bernoulli beam theory but where distortional displacements also are allowed. The formulation is based on a semi-discretization, which means that the cross-section is discretized into straight finite cross-section elements while the axial variation of the displacement functions is determined exact.

In the first assumption the Poisson's effect is included such that a coupling between axial and transverse strains exists. An energy formulation is afterwards used to set up a fourth order differential equation which was tried solved by expressing it by an eigenvalue problem. After eliminating all known singularities in the system, large numerical variations occurred in the differential equation and it could therefore not be solved correct in Matlab due to numerical errors. If the system could have been solved it, it would still lead to some issues that need to be considered. In the normal beam theory the variation of displacements out of plane along the beam, is described as the first derivative of the variation of the in plane displacements along the beam. For a pure axial displacement of the beam this would not be the case, when the Poisson's effect is taken into consideration. The formulation should therefore maybe be reconsidered and reformulated but have not been described further in this thesis since the differential equation could not be solved.

In order to be able to solve the homogeneous differential equation the energy formulation was expressed when the Poisson's effect was not included. Most of the singularities were again eliminated from the system and it was then solved by rewriting it into an eigenvalue problem. All natural mode shapes was then determined for the system and the corresponding analytical axial solutions were determined.

Different tests were then performed in order to check if the formulation included distortional modes as wanted. The test showed that the deformations not always were as expected when they were compared to simple hand calculations and results found in the commercial FEM-program ABAQUS. The tests though showed that distortional modes was included and existed. It was also found that the deformations due to shear did not contribute as much as expected which for some tests entailed to small deformations. The tests showed by analyzing the stresses that for a beam where the shear force should be constant along it, the shear stresses were not constant.

In order to check why the stresses were not as wanted the inner equilibrium equations were checked. By using equilibrium equations two independently differential equations could be determined. If these two equations were added together it turned out to be exactly equal to the differential equation solved to determine the mode shapes. It also showed that a higher order of interpolation functions must be used and may be the reason for incorrect stresses.

From the tests made it is given that the method can be used and do include distortional modes, but some further development is required in order to obtain correct shear deformations and stresses.

Sammenfatning

I dette kandidatspeciale er bjælkelementer med tyndvæggede tværsnit blevet analyseret. Dette er blevet analyseret da brugen af tyndvæggede elementer bliver mere og mere udbredt til anvendelse i byggesektoren pga. den høje styrke i forhold til materialeforbrug. Derfor er mere detaljerede og præcise beregninger nødvendige. Dette projekt tager udgangspunkt i artiklen "A Distortional eigenmodes and homogeneous solutions for semi-discretized thin-walled beams" skrevet af J. Jönsson og M.J. Andreassen og en videreudvikling er derfor udført, for at forsøge at medtage forskydningsdeformationer og ikke sætte begrænsninger ift. forskydning og udvidelse af tværsnittet. Dette gør det muligt at kunne inkludere Poisson effekten i beregningerne.

Teorien, der tages udgangspunkt i, er den generaliserede Euler-Bernoulli bjælketeori, men hvor tværsnitsdeformationer er medtaget, da de er vigtige for tyndvæggede profiler. Til analysen anvendes en semi-diskretisering, hvilket betyder, at tværsnittet opdeles i elementer, hvorved flytninger af dette løses ved en elementmetodeformulering, mens variationen hen over bjælkelementet løses analytisk.

Først forøges en analyse udført hvor Poisson effekten er inkluderet, hvilket vil sige, at der er en kobling mellem aksiale tøjninger og tværtøjninger. En energibetragtning anvendes til at opstille en fjerdeordens differentialligning, som er forsøgt løst som et egenværdiproblem. Det viste sig, at efter de kendte singulariteter var elimineret fra ligningssystemet, opstod der store numeriske forskelle i ligningssystemer. Dette medførte at Matlab lavede numeriske fejl når egenværdiproblemet blev løst, og resultaterne var derfor ikke korrekte. Hvis det havde været muligt at løse ligningssystemet, have yderligere overvejelser været nødvendige. I den normale bjælketeori er variationen af flytningerne på langs af bjælken beskrevet som den første afledte af variationen af flytningerne i tværsnittet. Ved en ren aksial forlængelse af tværsnittet vil dette dog ikke være tilfældet, når Poisson effekten er medtaget. Dette kunne medføre at formulering af differentialligningen skulle genovervejes, men er ikke yderligere overvejet her, da systemet ikke kunne løses.

For at blive i stand til at løse ligningssystemet er differentialligningen omskrevet til ikke længere at indeholde Poisson effekten, og dermed ignorere sammenhængen mellem aksiale tøjninger og tværtøjninger. Efter at have elimineret nogle af singulariteterne blev ligningssystemet igen løst ved et egenværdiproblem, og de naturlige formfunktioner kunne nu bestemmes. Derudover blev de aksiale løsningsfunktioner ligeledes bestemt for alle formfunktioner.

Forskellige tests blev udført, for at undersøge rigtigheden af løsningerne og om tværsnitsdeformationer indgik. Resultaterne af de forskellige test var ikke altid som forventet, når de blev blevet sammenholdt med simple håndberegninger og ABAQUS resultater. Det blev dog påvist, at tværsnitsdeformationer nu indgik som løsningsfunktioner og at resultaterne generelt er brugbare. Bidraget fra forskydningsdeformationer var dog ikke så stor som forventet, hvilket generelt har medført for små deformationer. Ud fra elementmetodeformuleringen blev spændinger også bestemt. Generelt var formen på spændingerne som forventet, men det blev påvist, at en konstant forskydningsspænding ikke blev opnået for en bjælke med konstant forskydningskraft.

De indre ligevægtsligninger blev anvendt for at se, om de var opfyldt og om dette var skyld i fejlen for beregningerne af spændinger. Ud fra de indre ligevægtsligninger kunne 2 uafhængige differentialligninger opskrives. Det viste sig at hvis disse 2 systemer blev lagt sammen, svarede det præcist til det system, der var blevet løst for at kunne bestemme formfunktionerne. Det viste sig også at fejlen i beregningerne for spændinger kunne skyldes interpolationsfunktionerne, og at interpolationsfunktioner af en højere orden kan være nødvendig.

Ud fra de udførte tests kan det konkluderes at denne metode kan anvendes til tyndvæggede tværsnit, da løsningen inkluderer tværsnitsdeformationer. En yderligere udvikling er dog nødvendig, for at opnå mere præcise resultater, og for at få forskydningsdeformation og spændinger korrekt bestemt.