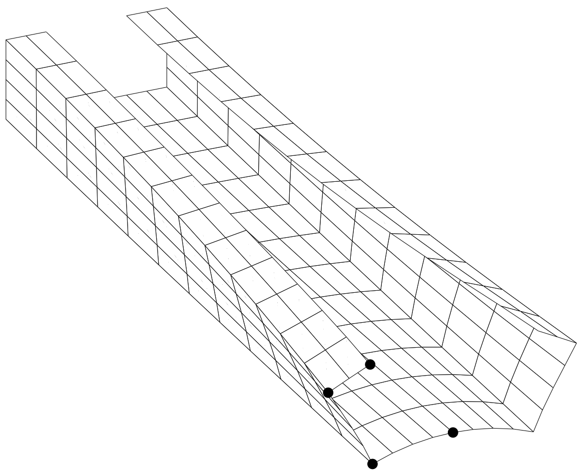
Advanced Beam Elements with Distorting Cross Sections



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MSc Thesis

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Preface

This Master's Thesis is a part of a research project on distorsional behaviour of thin-walled structural elements lead by Professor Jeppe Jönsson and Associate professor Michael Joachim Andreasen at the Department of Civil Engineering at DTU. The research has been performed in cooperation with Lotte Braad Sander while the reporting part has been handled individually. The Matlab program and manual have been developed jointly by both parties.

The thesis primarily relates to the work done in the articles [Jönsson & Andreasen, 2010] and [Jönsson & Andreasen, 2012]. References to these articles are therefore as a main rule avoided except for citations or when it is found necessary to emphasize the references. Parts of the work performed by previous student Michael Teilmann Nielsen in his Master's Thesis from 2012 is also further elaborated.

In order to get an idea of how the *Generalized Beam Theory* (GBT) is implemented in Matlab, a HTML visualisation of the main Matlab file has been added in Appendix 11. Unfortunately the HTML visualisation is at the expense of vector graphics, but the clarity of the script is preferable.

This Master's Thesis comprises 30 ECTS credits and has been submitted on February 22, 2013.

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Abstract

The use of thin-walled members is increasing as are the attention and need for more detailed calculations comprising the distorsional behaviour of thin-walled members. In order to include the distorsional behaviour in the calculations it is necessary to extent the classic Euler-Bernoulli theory. The formulation and implementation of an advanced beam element in a finite element context which facilitate both flexural, torsional and distorsional behaviour is considered. The formulation of the beam element relates to the work performed in [Jönsson & Andreasen, 2010] and [Jönsson & Andreasen, 2012]. Thus this Master's Thesis consists of; a Matlab program based on *generalized beam theory* (GBT), a manual for the program and the present thesis on the theory and research from which the Matlab program is developed.

The formulation and development of the advanced beam element is based on a cross-sectional analysis, where the cross-section is assembled from straight beam elements which hold both in-plane and one out-of-plane degrees of freedom. A displacement field based on Bernoulli theory is introduced and the potential energy is established through simple constitutive relations. It is highly desirable to include *Possion* in the constitutive relations but the inclusion produces a coupling term which complicates the GBT-formulation wherefore the coupling term is neglected.

From a virtual work formulation the first variation of the potential energy is investigated by the introduction of a virtual displacement field and by means of partial integration leading to the governing differential equations (GDE). It is found that the GDE can also be derived from inner equilibrium equations leading to two sets of GDEs, which when added are identical to the ones found from the virtual work formulation.

From the GDE a generalized eigenvalue problem is constructed from which the convential beam and distorsional deformation modes are identified. The axial variation functions for each cross-sectional deformation mode are identified and special attention is given to the conventional beam deformation modes and the related axial variation functions. Hence the theory is denominated semi-discretization as the cross-section is discretized while the solution along the beam is represented by analytical functions. Based on the axial variation functions and the cross-sectional deformation modes, the distorting beam element formulation is developed.

Displacement and stresses are determined using general finite element (FE) theory. Due to the chosen displacement field and interpolation functions it is possible to split the stresses into a membrane part and a bending part. Deformation and stress tests are performed and the results are compared to results obtained from [Nielsen, 2012] and the commercial FE-program Abaqus whereupon conclusions are drawn on the basis of the performance of the beam element.

Resumé

Brugen af tyndvæggede profiler øges, ligesom behovet for mere detaljerede beregninger, som omfatter forvrængningen/formændringen af de tyndvæggede profiler stiger. For at kunne inkludere forvrængningsdeformationen i beregningerne er det nødvendigt at udvide den klassiske Euler-Bernouille teori. I dette speciale udvikles og implementeres et avanceret bjælkeelement i en finite element kontekst, hvor både bøjning, rotation og forvrængning er mulig. Konstruktionen af bjælkeelementet relaterer sig til arbejdet udført i [Jönsson & Andreasen, 2010] og [Jönsson & Andreasen, 2012]. Følgelig består dette kandidatspeciale af; et Matlab program baseret på en generaliseret bjælke teori (GBT), en manual for programmet og nærværende afhandling, som indeholder teori og forskning, hvorfra Matlab programmet er konstrueret.

Formuleringen og udviklingen af det avancerede bjælkeelement bygger på en tværsnitsanalyse, hvor tværsnittet er opbygget af lige bjælkeelementer, som har både frihedsgrader i planen og ud af planen. Et flytningsfelt baseret på Bernoulli-teori introduceres, og den potentielle energi kan bestemmes gennem simple konstitutive relationer. Som udgangspunkt er det forsøgt at inkludere *Possion* i de konstitutive ligninger, men dette medfører et koblingsligsled, som besværliggør GBT-formuleringen, hvorfor der ses bort fra dette koblingsled.

Ved brug af virtuelt arbejdes princip undersøges den første variation af den potentielle energi ved at introducere et virtuelt flytningsfelt samt partiel integration, hvorved de styrende differentialligninger (SDL) findes. Ydermere er det fundet, at SDL kan findes ud fra indre ligevægtsligninger, hvor disse SDL er identiske med dem, der er fundet ved brug af virtuelt arbejdes princip.

Fra SDL konstrueres et generaliseret egenværdiproblem, hvorved de konventionelle bjælke og forvrængede formfunktioner kan bestemmes. Efterfølgende bestemmes de respektive aksiale løsningsfunktioner, relateret til tværsnittets formfunktioner, og der sættes særlig fokus på identifikationen af de konventionelle bjælke-formfunktioner og de relaterede aksiale løsningsfunktioner. Følgelig benævnes teorien semi-diskretisering da tværsnittet er diskretiseret mens løsningen langs bjælkeelementet beskrives ved analytiske funktioner. Formuleringen af bjælkeelementet sker gennem tværsnittets formfunktioner og de aksiale løsningsfunktioner.

Deformationer og spændinger bestemmes ved brug af generel finite element (FE) teori, og grundet det valgte flytningsfelt og de valgte interpolationsfunktioner, er det muligt at opdele spændingerne i en membrandel og en bøjningsdel. Deformations- og spændingstest udføres, og resultaterne sammenlignes med resultater fra [Nielsen, 2012] og resultater fra the kommercielle FE-program Abaqus, og der konkluderes på bjælkelementets præstation.