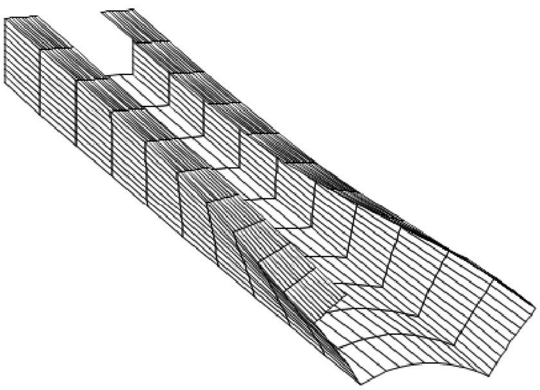
Generalized Beam Theory

Distortional mechanics of thin walled structures using an energy and equilibrium approach



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Preface

This Bachelor thesis comprises 15 ECTS credits and is a part of a research project on the distortion of thin walled structural elements lead by Professor Jeppe Jönsson and Adjunct Michael Joachim Andreassen at the Department of Civil Engineering at DTU. The project involves a report and a MATLAB program in which the theory is implemented.

The first part of the thesis involving the energy approach is made in cooperation with Simon Larsen, Andreas Hoffmann, Anders Hejnfelt and Jesper Pedersen and is based on a Ph.D. thesis by M.J. Andreassen [1]. The cooperation included exchange of ideas in relation to the programming part and understanding the theory. The report and program are though made individually.

I would like to thank Adjunct Michael Joachim Andreassen for the advice on the programming part and help through the project.

A special thank goes to my main supervisor Professor Jeppe Jönsson for the guidance and advice. It has been enjoyable to see his interest and gain a part of his extensive research experience in this field.

Abstract

Today thin walled structures comprises a growing part of engineering construction with diverse areas of application ranging from wind turbines, aircraft, bridges, ships to industrial building and warehouses. Because of the high slenderness of these structures, deformations in cross sections have to be considered. Thus it is necessary to include distortion which is not covered by the classical beam theory and whose analysis requires very time consuming calculations, due to the large number of degrees of freedom.

The present thesis deals with distortional mechanics of thin walled beam elements accompanied by a semi-discritization method where the cross section is discritized. Two formulations originating in energy and equilibrium considerations are applied. Based on these considerations the governing distortional homogeneous differential equations are constructed and solved as an eigenvalue problem, in which the natural deformation modes represents the eigenvectors and the axial variation of the modes represents the eigenvalues.

In the first part of the thesis an energy formulation is set up. The extended beam theory incorporating the distortion phenomenon is developed by assuming a set of orthogonal displacement fields. The displacements along the beam are expressed as the product of the cross-section displacement function and the axial variation function. Based on the derived strains field, the elastic potential energy is formulated.
A semi-discritization method is applied and the potential energy equation is formulated in a discritized form. Minimizing the potential energy leads to a stationary state from which a homogeneous differential equation system arises. In order to condense this eigenvalue problem the singularities are systematically identified and eliminated. Due to assuming constitutive relations without Poisson, the constraints related to constant wall width and pure axial extension due to shear, are enforced. The distortional differential equation systems are finally solved by assuming an exponential axial variation of the local deformation modes.
In the second part, an equilibrium formulation is applied. An alternative axial displacement fields are used and the equilibrium equations of the cross section are expressed in strong form. By applying the principle of virtual work, the weak form is obtained. This form is specified by assuming linear elastic behaviour with Poisson and defining the displacement field as a product of the local displacements and the axial variation. To find the strong form of the equilibrium equations along the beam, the energy is minimized whereby stationary state is obtained. The arising homogeneous differential equation is attempted to be solved by neglecting the constraints as a consequence of the Poisson effect. However it has not been possible to get through the equations, the reason why a comparison between the two approaches is not conducted. Developing a solution method and investigating the continuity requirements in the equilibrium part is therefore highly desirable.

Resume

I dag udgør tyndvæggede konstruktioner en vigtig del af bygningsbranchen med forskellige anvendelsesområder der strækker sig fra vindmøller, flymaskiner, broer, skibe til industribygninger og lagre. Grundet disse konstruktioners slankhedsforhold, bør lokale deformationer i tværsnittet betragtes. Derfor er det nødvendigt at inkludere tværsnitsdeformationer som ikke bliver berørt af den klassiske bjælketeori, og hvis analyse beregningsmæssigt er tidskrævende grundet det store antal af frihedsgrader.

Denne opgave omhandler derfor en konstruktionsteori der tager højde for tværsnitsdeformationer og er understøttet af en semidiskritiseringsmetode, hvorved tværsnittet diskritiseres i stedet for at diskritisere på langs. To formuleringer bundet i energi- og ligevægtsbetragtninger anvendes. Baseret på disse betragtninger udtrykkes den styrende homogene differentialligning og løses ved at betragte et egenværdiproblem, hvori de naturlige deformatonsformer repræsenterer egenvektorerne mens den aksiale variation af formerne repræsenterer egenværdierne.

Den første del af opgaven omhandler energiformuleringen. En udvidet bjælketeori der omfatter tværsnitsdeformationer udvikles ved at antage et sæt af ortogonale flytningsfelter. Flytningen langs med bjælken udtrykkes som produktet af den lokale tværsnitsdeformationsfunktion og den aksielle variationsfunktion. Med udgangspunkt i de afledte tøjningsfelter, formuleres den elastiske potentialenergi. Semi-diskritiseringsmetoden anvendes og potential energiligningen formuleres i en diskritiseret form. Ved at minimere energien i systemet opnås en stationær tilstand hvorfra den homogene differentialligningssystem fremkommer. For at gøre omfanget af egenværdiproblemet mindre, identificeres samt elimineres singulariterne på systamtisk vis. Grundet antagelsen om de konstitutive relationer uden Poisson forhold indføres restriktioner. Endeligt løses den styrende differentialligningssystem ved at antage en eksponentiel aksial variation af de lokale deformationsformer.

Den anden del af opgaven omhandler ligevægtsformuleringen. Et alternativt aksial flytningsfelt anvendes og ligevægtsligningerne for tværsnittet opstilles på stærk form. Ved brug af virtuelt arbejde, fås den svage form af ligningerne. Disse specificeres ved at antage en lineær elastisk matrialeopførsel med Poisson forhold samt definere flytningsfelterne, som produktet af de lokale tværsnitsdeformationer og den aksiale variation langs med bjælken. For at finde den stærke form af ligevægtsligningerne på langs, minimeres energien hvorved en stationær tilstand opnås. Den fremkommende homogene differentialligningssystem er forsøgt at blive løst ved at negligere restriktionerne som følge af Poisson effekten. Det har dog ikke været muligt at komme igennem ligningerne, hvorfor en sammenligning mellem de to tilgange (ligevægt og energi) ikke er blevet udført. Det er derfor anbefalet at man finder en løsningsmetode samt undersøger randledene i den svage form.